Response to Netz
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there is something else: Greek mathematics is a prime example of discovery. The mathematician, after all, deals with *heuresis*, with the finding of new results. More than that, he finds that which was hidden in plain sight but which, for sight alone, would have to remain hidden and could be made seen only through the mind's eye. Thus the pleasure of doing mathematics is built on a faith in the ability of humans to find knowledge that goes beyond sense perception alone and to express such knowledge in human language.

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RESPONSE TO NETZ

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The question put forward by Reviel Netz, "What did Greek mathematicians find beautiful?," carries with it a larger question about motivations. Even though it is difficult to identify other clear motivations—social, institutional, professional, and so on—for Greek mathematicians' activities, there remains the notion that they found something beautiful about what they did and that the resulting pleasure is what spurred them on in their mathematical pursuits.

Such an inquiry is fraught with potential missteps, and the sorts of complications that one encounters are nicely set forth in Wilbur Knorr's treatment of the issue.[1] The problems stem in large part from the use of the indirect tradition, specifically the philosophically influenced treatments of mathematics. Netz avoids such issues by restricting his use of evidence to the written products of Hellenistic mathematicians and by deliberately leaving out contemporary or near-contemporary commentary by nonmathematicians.[2] In particular, the philosophical treatments by Plato and Aristotle draw us into a metaphysical inquiry, and we are led to think that perhaps there is something beautiful about mathematical objects and the relations that hold between them, and we begin to wonder what these objects are—forms, abstractions, intermediates, or something else—as well as what they are like. But if Netz is correct in locating the beauty of Greek mathematics in the act of discovery, in the recognition of heretofore unseen relations between mathematical objects, then the preoccupations of the philosophers will have taken us quite far from the actual motivations and interests of the mathematicians.

It is thus better methodologically to exclude such external treatments of mathematics from this inquiry. Yet once some conclusions have been reached (even if they are in large part negative and only tentatively positive, as Netz acknowledges), we can make use of them when reexamining the third-party evidence. That is to say, an improved understanding of the aesthetics of ancient mathematics, developed apart from the philosophical evidence, can make some meaningful contribution to our reading of ancient philosophy. This re-

1. Knorr 1986, chap. 1.
2. Netz, p. 426, n. 1 above.

sponse will suggest some ways in which the analysis at hand offers additional insight into and context for Plato's interest in and use of mathematics, specifically for his appeal to mathematics and mathematical method in the *Republic*.

Netz argues that the aesthetics of Greek mathematics depends in large part on a tension between the visible and the invisible. The mathematician "finds that which was hidden in plain sight but which, for sight alone, would have to remain hidden and could be made seen only through the mind's eye."[3] The tension involves both text and diagram, but here I will leave aside issues of the text and instead focus on the representational nature of diagrams. In particular, I want to foreground the way in which diagrams contrast the visual and the structural. On this basis I will then discuss some consequences of this contrast for our reading of *Republic* 6–7.

Greek mathematical diagrams, as visual representations, cannot be characterized as beautiful. They illustrate the topology of the mathematical objects that they represent, and they do so in a deliberately schematic way. Such diagrams do not, so far as we can tell, attempt to bring out the beauty of the objects that they represent: as Netz notes, "[w]hat is typical of ancient mathematical diagrams is that they deliberately avoid the visual evocation of a spatial object. Instead, they are schematic, almost conceptual maps of the topological pattern of overlap and intersections within a configuration."[4]

As Netz puts it, Greek mathematicians chose "to make the visual representation subservient to textual encoding."[5] And here the interplay between text and diagram helps us to realize that the diagram, though visual insofar as it is apprehended by sight, does not constitute a visual representation at all. Rather, it is a *structural representation*; it communicates the essential points about how mathematical objects in a particular configuration are related, and no more. And such relations are precisely what one ought to "see" when doing mathematics. The diagram is a representation, true, but—to use the distinction highlighted in Aryeh Kosman's paper—it is not a "mere representation," but rather the sort of representation that could truly be called a "presentation of being."[6]

In consequence, we should not say that a diagram works in spite of its visual inadequacy; rather, it works by virtue of its anti-visual nature. There is nothing wrong with a visually ugly or distorted diagram, so long as it clearly communicates the relevant topological features of the mathematical configuration. Such distortions can be taken to the extreme, as when the triangle is used to represent a parabolic segment, or curved lines tangents to a circle.[7] Visual

3. Netz, p. 444 above.
4. Netz, p. 431 above.
5. Netz, p. 433 above.
6. Kosman, p. 355 above.
7. See Netz, p. 442, fig. 1 above, for a parabolic segment represented with straight lines. Several diagrams in the tenth-century Archimedes Palimpsest use curved lines to represent tangents to a circle (and by the principle of *lectio difficilior*, it is not inconceivable that this sort of representation goes back to Archimedes' own hand). See, e.g., the figure to *On the Sphere and Cylinder* I.32, reproduced in Netz 2004, 142. The Palimpsest's version of this figure can be seen in the digital images to folio 150r, available at http://archimedespalimpsest.net/Data/150r-153v/. See in particular the upper left part of 150r-153v_Arch58r_Sinar_pseudo_sharpie.jpg (accessed July 2010).

semblance and metrical accuracy are irrelevant to a diagram's use. Through their simplifications and distortions, Greek mathematical diagrams exploit the contrast between the visual and the structural.

It is with these features of the diagram in mind that I would like to examine the Divided Line passage at the end of *Republic* 6. This is the second of the images offered by Socrates to convey some sense of what "the good" is. He famously explains the third segment of the Divided Line, *dianoia*, by pointing to the methodology of those who deal with geometry and related subjects. Here he highlights two aspects of their mathematical practices: (1) they base their inquiries on hypotheses; and (2) they make use of visible forms, taking them as images of something else and conducting their inquiries for the sake of those represented originals. While there is much dispute over what is meant by "hypothesis" here,[8] it is clear that the visible forms used by mathematicians are the figures that appear in diagrams.

Now, even though *dianoia* operates within the realm of the intelligible, it is subordinate to dialectic (which is the only means by which one can apprehend the good, and which does not rely on hypotheses or images), and so one might be tempted to read criticism into Socrates' description of it. But not only is such a reading not borne out by the wording of the passage,[9] it is also inconsistent with the entire thrust of the curriculum presented in *Republic* 7. The prevailing concern throughout the curriculum passage is the upward conversion of the soul toward the intelligible realm. Each subject is evaluated for inclusion according to its ability to foster such a conversion. It is a theme that Plato stresses throughout, even to the point of having Glaucon mistakenly praise the astronomer's viewing of the heavens in order that Socrates can reprimand him for confusing the upward gaze of the eyes with that of the intellect.[10]

When we think of diagrams and their use, all this makes sense. In the process of working out or presenting a proof, the geometer looks at visual representations of, for example, the square and its diagonal, and he refers to them through the various steps of reasoning. He cannot, however, simply take the figures as they are, visually. The diagram may show that there is a four-sided figure ΑΒΓΔ with a diagonal running from point Β to point Γ. But because the diagram is metrically imperfect (and necessarily so, for it is something perceived by the eyes), it is up to the geometer to assert that all four sides of ΑΒΓΔ are of equal length and perfectly straight, that its four angles are all right angles, and that ΒΓ is a perfectly straight line.[11] Though the geometer works with the drawn square, he relies on the properties of an actual square—only then would he be able to demonstrate, for example, that the diagonal ΒΓ is incommensurable with the side of ΑΒΓΔ.

8. Various possibilities are discussed in Netz 2003b.

9. As argued in Burnyeat 1987, 218–19. Netz (2003b, 304–6) takes it as given that Socrates criticizes the mathematicians' reliance on hypotheses, but nevertheless maintains that he in turn praises their way of using diagrams.

10. *Resp.* 528e4–529c3 and Shorey 1935, ad loc. (All translations are from this edition of the text.)

11. Indeed I would suggest that such features—the square-ness of ΑΒΓΔ and the straight-line-ness of ΒΓ—are among the "hypotheses" that the geometer sets down when conducting his inquiry.

This way of interacting with the diagram is consistent with Netz's analysis, and it fits with the procedure that Socrates ascribes to geometers: they use visible figures, but they think about the originals of which the visible figures are images, and they do all this for the sake of those originals (510d). But given the radically anti-visual, structural nature of diagrams, we can identify another, more basic way in which they foster the upward conversion of the soul. In the discussion of arithmetic, Socrates points to the importance of "contradictory perceptions" as a first step in the awakening of thought and reflection. The production of a contradictory perception "provokes thought toward inquiry" (523b). If we accept that mathematical diagrams going back to the late fifth and early fourth century had the same features as Netz describes, it seems perfectly reasonable to assume that Plato recognized the visual discrepancy between how a diagram looks and what it represents. So then, the square in the diagram, too, could be productive of contradictory perceptions. It both is and is not a square, in that it undoubtedly projects a look of "square-ness" even though it at the same time exhibits uneven sides, perhaps slightly bent, which do not meet at clean right angles.

We can take this phenomenon one step further. To return to the triangle that represents a parabolic segment:[12] the triangle in the diagram looks like a triangle, even with its imperfections, yet the mathematical configuration tells us that this figure has been set down as a parabolic segment. Whatever reliance on the metrics of the figure we had retained must now be completely abandoned. The parabolic curve (depicted as two sides of a triangle) falls completely within the curve of the enclosing circle, and that is all that is needed for this part of the diagram to work. The use of diagrams, then, can to varying degrees help to draw one away from a reliance on visual perception, and presumably this mechanism for developing abstract thinking is in large part what Plato has in mind when he has Socrates use geometers' methods to explain the third section of the Divided Line and include geometry as one of the studies in the curriculum.

By now it is clear, I hope, how the contrast between the visual and the structural plays a central role both in the historical use of diagrams and in Plato's appeal to them in the *Republic*. Their privileging of the structural, as I have suggested, appropriately qualifies them to be called "presentations of being." By extending the reading of *dianoia* to include nonmathematical cases, we can identify some additional consequences of this structural mode of representation.

The manner in which the upper part of the Divided Line is presented leaves open the possibility of making this move. Socrates' initial description of *dianoia* points to the use of images and the reliance on hypotheses, but not within the more limited context of mathematics. It is only after Glaucon finds the description too opaque that Socrates resorts to a description of mathematical methods. A bit further on, we are told to liken the Cave image that follows to the preceding images of Sun and Line.[13] In doing so we see that

12. Netz, p. 442, fig. 1 above.
13. Here I follow the standard reading of *Resp*. 517a8–b3. See Burnyeat 1987, 228 n. 38.

the third stage of the prisoner's ascent corresponds nicely with the third section of the Line. Once the prisoner has been dragged out of the cave and into the light of the outside world, he must use shadows and reflections as a means of apprehending the things out there, and only later will he be able to directly look at the things themselves.[14] Although the precise ontological status of the objects involved is less than clear, the methodology should be entirely familiar: Here we recall the geometer's practice of using images (the figures in diagrams) to better understand the intelligible realm (mathematical relations).

What would be involved in applying this methodology to nonmathematical cases? Roughly put, and by analogy with the geometrical case: one posits "figures" or objects of some sort in a particular configuration and assigns relevant qualities to them. Taking this configuration as a starting point, one then analyzes or manipulates the configuration in order to see what further properties can be deduced from it (as with theorems), or what can be developed from it (as with problems). And indeed the present images of Sun, Line, and Cave seem to provide us with three examples of nonmathematical *dianoia*. Socrates explicitly calls the Sun and Cave εἰκόνες,[15] and the Line's very construction calls to mind a mathematical diagram. More significantly, before launching into these images, Socrates says that he will talk about "what seems to be the offspring of the good and most nearly made in its likeness" (506e2–3). That is, the images are somehow produced from the good, and in some way they evoke or represent its being. And by studying them we can in some way attain a greater understanding of what is meant by "the good," even if what we get does not count as secure knowledge.

That the images that appear throughout the *Republic* are instances of nonmathematical *dianoia* is not a new suggestion. But I would like to draw attention to certain features of Plato's image making in light of what we have seen about the practices surrounding mathematical diagrams. A recurring theme thus far is how the surface features of represented objects can be radically different from their structural features. The important features of a parabolic curve cannot be conveyed by its visual appearance, for its ability to produce proportions between squares and lines—to effect a dimensional conversion[16]—ought to be apprehended by and exercised through the intellect. As such, a straight line will suffice for its representation (and it offers the added benefit of providing greater structural resolution if the image is small and cramped). In like manner with mathematical diagrams, nonmathematical images can vary in their modes of representation. They can evoke properties based on appearance or those based on being—in other words, they can focus on visual representation or on structural representation.

14. *Resp.* 516a1–8, with a recapitulation at 532b6–c4. The shadows and reflections are a deliberate echo of 510e2–3 (in the description of *dianoia*) and, before that, 510a1–3 (in the description of *eikasia*).

15. The term εἰκών is used of the Sun image at 509a9 and of the Cave image at 515a4, 517a8, 517d2, and 533a2.

16. Netz, p. 434 above.

A focus on the structural can account for why the images of Sun, Line, and Cave look so different, for each can only deal with so many aspects of the good, and each chooses to foreground different aspects of it; yet there are still enough commonalities for one to recognize their kinship as "offspring of the good." It also helps to explain why some images look so strange. Earlier in Book 6 Socrates elucidates the present condition of philosophers through the Ship of State image (this, too, is called an εἰκών, 487e4–5). Socrates apologizes for its peculiarity and compares himself to a painter portraying goat-stags and other such creatures (488a2–6). He is compelled to assemble what amounts to a strange image, at least with regard to men on a ship. But the image is not about the pilot's condition on the ship; rather, it really describes the philosopher's condition in the city. In like manner it is initially jarring to see a curved tangent line. But it is not really a curved line; it is a straight line. And its representation via the curved line is there to more clearly reflect the line's relationship to the circle. (Similar workings can be observed, for example, in the Noble Lie and the Myth of Er.)

In conclusion I would like to return to the issue of pleasure, by way of the issue of strangeness. Netz locates the aesthetics of the diagram "in the pleasurable act of interpretation, taking a mere linear network and 'reading off' its intended spatial significance."[17] So too in the case of images, except that instead of a linear network we get whatever sort of representation, no matter how strange, is best suited to conveying the underlying structure of the represented object or situation. The prime example here is the Cave: the image is deeply bizarre and unsettling, and yet it invites us to examine its meaning. In a short exchange early on in the passage, Plato makes sure that we have been hooked in by it: " 'A strange image you speak of,' [Glaucon] said, 'and strange prisoners.' 'Like to us,' I said . . ." (515a). Elsewhere Plato acknowledges the attractiveness of mathematical studies. In his discussion of geometry and stereometry, Socrates acknowledges that, in spite of the various obstacles to these studies, they "force their way by their inherent charm." To this, Glaucon replies, "It is true that they do possess an extraordinary attractiveness and charm" (528c6–d1). If, as Netz concludes, the aesthetics of Greek mathematics is tied to an epistemology of discovery, to the challenge and pleasure of decoding the meaning that underlies an opaque representation and an opaque text, then this gives us, I think, a helpful way of better understanding what Plato found so compelling about mathematics and "mathematical" ways of reasoning.

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17. Netz, p. 433 above.